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BRIEF COMMUNICATION

SEDIMENT COMPOSITION DUE TO SETTLING OF PARTICLES OF DIFFERENT SIZES

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1. INTRODUCTION

Sedimentation of particles of nonuniform size has already been studied by several authors (Smith 1965, 1966; Lockett & Al-Habbooby 1973, 1974; Masliyah 1979; Batchelor 1982; Batchelor & Wen 1982; Davis, *et al.* 1982; Greenspan & Ungarish 1982). So far, however, little attention has been paid to the end-product of the sedimentation process, i.e. the composition of the sediment at the bottom of the vessel (cf. figure 1). Greenspan & Ungarish (1982) did determine the volume fractions and particle distribution functions in the sediment but their analysis rests on the assumption that the voidage (or the total particle volume fraction) in the sediment is a given constant which does not depend on the particle size distribution. However, this assumption is not always justified. Experiments (Furnas 1929, Jeschar 1964) showed that the voidage in a packed bed can be drastically changed by increasing the amount of smaller particles in the mixture. The obvious reason for this effect is that sufficiently small particles can partly fill the gaps between the larger particles, cf. also the analytical investigations by Hudson (1949).

In what follows we shall apply available semiempirical correlations for packed bed voidage (Jeschar 1964) to batch sedimentation of particles of two different sizes in vessels with vertical walls. The relative motion of particles and liquid is described by means of a drift-flux model (Masliyah 1979) which is a simple generalization of the well-known approximation due to Richardson & Zaki (1954). The analytical predictions are confirmed by experimental data.

2. SEDIMENT COMPOSITION RELATIONS

Consider a packed bed of spherical particles with diameters d_α and d_β ($>d_\alpha$). Let α_s and β_s be the fractions of total volume occupied by the particles with diameters d_α and d_β, respectively. Note that the void fraction is $1 - (\alpha_s + \beta_s)$. In case of uniform particle size the void fraction in the packed bed would be equal to a constant value ϵ_u (approximately 0.32 according to our experiments, cf. below). For particles of different sizes, however, the void fraction depends on the diameter ratio d_α/d_β and the volume fraction ratio α_s/β_s.

Based on a theoretical investigation of the limiting case $d_\alpha/d_\beta \to 0$, Jeschar (1964) was able to correlate his own experimental results as well as previous results due to Furnas (1929) for $d_\alpha/d_\beta \neq 0$. Jeschar's correlation can be rewritten as follows:

$$\alpha_s = (1 - \epsilon_u)(1 - \beta_s) - \epsilon_u\beta_s(d_\alpha/d_\beta)^n, \quad 0 \le \beta_s \le 1 - \epsilon_u, \qquad [1]$$

with

$$n = 0.6 \sin\left(\frac{\pi\alpha_s}{\alpha_s + \beta_s}\right). \qquad [2]$$

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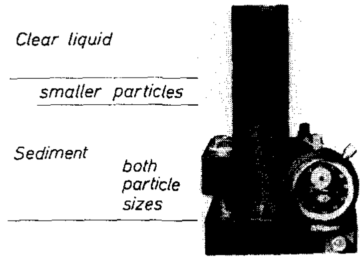

Figure 1. Sediment of white (d_α = 125 μm) and blue (d_β = 496 μm) particles obtained by batch sedimentation.

Figure 2a (solid line) shows results of [1] and [2] for the diameter ratio $d_\alpha/d_\beta = 0.25$. Of course, the void fraction would differ from that one for uniform particle size (dashed line) the more the larger the difference in particle size. Note also that for very small values of d_α/d_β [1] predicts a regime of (rather small) values of α_s in which β_s remains constant although α_s changes. Regarding figure 2b cf. section 4. The experimental data given in figure 2 are discussed in section 5.

3. DRIFT FLUX AND JUMP RELATIONS

For a given diameter ratio d_α/d_β, [1] is one relation for two unknowns, α_s and β_s. This expresses the fact that the concentration of, say, the larger particles in the sediment depends on their concentration in the suspension above the sediment. This coupling between sediment composition (including sediment voidage) and suspension composition is governed by the conservation of mass (or volume) at the interface between sediment and suspension.

Applying the well-known drift-flux concept (cf. Wallis 1969) to a mixture of a fluid (subscript f) and two species of particles (subscripts α and β, respectively), the drift-fluxes $j_{\alpha f}$ and $j_{\beta f}$ are defined as the volume flux densities of components α and β, respectively, relative to the volumetric average velocity (which is zero in case of batch sedimentation). Thus

$$j_{\alpha f} = j_\alpha - \alpha j_t = \alpha(1 - \alpha)v_{\alpha f} - \alpha\beta v_{\beta f} = -j_{f\alpha}; \tag{3a}$$

$$j_{\beta f} = j_\beta - \beta j_t = \beta(1 - \beta)v_{\beta f} - \beta\alpha v_{\alpha f} = -j_{f\beta}. \tag{3b}$$

Here j_α, j_β are the volume flux densities of phases α and β, respectively, j_t is the total volume flux density, and $v_{\alpha f}$, $v_{\beta f}$ are the relative velocities of, respectively, phases α and β with respect to the fluid.

Extending the widely used correlation due to Richardson & Zaki (1954) to nonuniform particle size, Masliyah (1979) suggested the following relation between the relative velocities and the void fraction ($\epsilon = 1 - \alpha - \beta$) in hindered settling:

$$v_{\alpha f} = \epsilon^m g\,(\rho_\alpha - \bar{\rho})d_\alpha^2/18\mu_f; \tag{4a}$$

$$v_{\beta f} = \epsilon^m g\,(\rho_\beta - \bar{\rho})d_\beta^2/18\mu_f, \tag{4b}$$

where g is the gravity constant, μ_f is the fluid viscosity, and ρ_α, ρ_β, ρ_f, $\bar{\rho}$ are, in this order, the mass densities of phase α, phase β, fluid and suspension, with $\bar{\rho} = \alpha\rho_\alpha + \beta\rho_\beta + \epsilon\rho_f$.

A suitable value of the constant m is, according to Masliyah (1979), $m = 2.7$, but the value of m does not matter for the present analysis. In fact, replacing ϵ^m in [4a, b] by any function of ϵ would lead to the same result for the sediment composition, cf [6].

In the sediment, fluid and particles are at rest. Thus conservation of mass of the incompressible phases requires that the following relation be satisfied at the sediment/

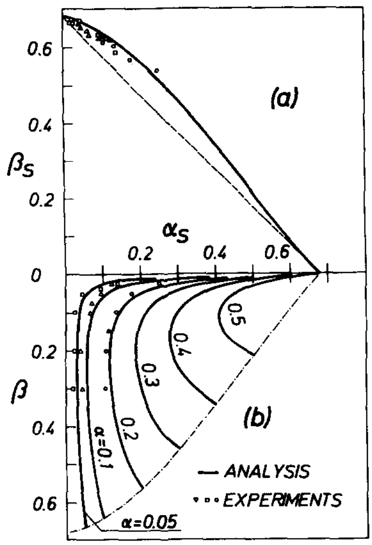

Figure 2. Sediment composition with spherical particles of two sizes; diameter ratio $d_\alpha/d_\beta = 0.25$. (a) Volume fraction relation in the sediment. (b) Sediment volume fraction of smaller particles, as a function of particle volume fractions in the suspension, α and β. $\rho_\alpha/\rho_f = \rho_\beta/\rho_f = 2.31$.

suspension interface:

$$\frac{j_{\alpha f}}{\alpha_s - \alpha} = \frac{j_{\beta f}}{\beta_s - \beta}. \qquad [5]$$

The terms on either side of [5] are equal to the propagation velocity of the interface with respect to the sediment at rest.

4. RESULTS

Expressing the drift fluxes in [5] in terms of the volume fractions with the help of [3] and [4], one obtains

$$[1 - \beta(1 + F)]\alpha_s - [F - \alpha(1 + F)]\beta_s = \alpha - \beta F, \qquad [6]$$

where

$$F = \frac{\alpha(\rho_\alpha - \bar{\rho})}{\beta(\rho_\beta - \bar{\rho})}\left(\frac{d_\alpha}{d_\beta}\right)^2. \qquad [7]$$

Note that the term ϵ^m (describing the hindered settling) as well as the gravity constant and the fluid viscosity have dropped out.

For given particle volume fractions α, β in the suspension immediately above the sediment/suspension interface, [1] and [6] are a system of two equations for α_s and β_s, i.e. the particle volume fractions in the sediment. Note that the exponent n in [1] contains the unknowns, cf [2]. Furthermore, it has to be taken into account that there is an upper limit for β_s, cf [1]. The system of [1] and [6] can be solved iteratively. Some numerical results are shown in Fig. 2b. By means of combining the results given in figure 2b with the sediment composition relation given in figure 2a, any two unknowns of the quadruple α, β, α_s, β_s can easily be determined if the other two values are given.

5. EXAMPLE AND COMPARISON WITH EXPERIMENTAL DATA

As an example, batch sedimentation of particles of two different sizes is considered. The initial concentrations are assumed to be sufficiently low to avoid the existence of a

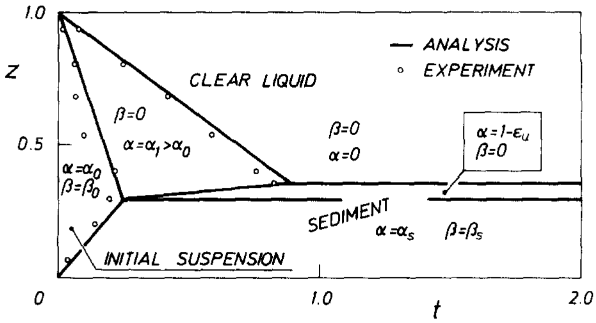

Figure 3. Batch sedimentation of two particle species in the space-time diagram. Vertical coordinate z referred to the vessel height, time t referred to the vessel height divided by the terminal velocity of a single particle of the smaller species. Given suspension: $d_\alpha/d_\beta = 0.25$, $\rho_\alpha/\rho_f = \rho_\beta/\rho_f = 2.31$, $\alpha_0 = 0.05$, $\beta_0 = 0.2$. Settling process: $\alpha_1 = 0.067$ (analysis). Sediment composition obtained: $1 - \epsilon_u = 0.681$ (experiment); $\alpha_s = 0.029$ (analysis), 0.023 (experiment); $\beta_s = 0.673$ (analysis), 0.674 (experiment).

continuous variation of the concentration in the suspension, cf. Wallis (1969). Rather the particle volume fractions change in discontinuities (interfaces) only, and the space-time diagram of the process is as shown in figure 3. The particle volume fractions in the lower part of the sediment, where both species are present, are obtained from figure 2. Due to the different settling rates of particles of different sizes, the upper layer of the sediment contains particles of the smaller species only.

To check the validity of the assumptions, experiments were performed with glass beads ($\rho_\alpha = \rho_\beta = 2790$ kg/m^3, $d_\alpha = 125$ μm, $d_\beta = 496$ μm) in a mixture of 95% glycerine and 5% water ($\rho_f = 1208$ kg/m^3, $\nu_f = \mu_f/\rho_f = 2 \times 10^{-5}$ m^2/s), cf. figure 1. The smaller particles were blue and could easily be distinguished from the larger, white particles. The size of the apparatus was 41 × 25 × 300 mm (length × width × height). The position of the interfaces was observed as function of time, and the composition of the sediment was determined by measuring the height of the two layers (mixture of both particle species at the bottom, smaller particle species on top, cf. figure 1) and making use of the total mass balance. The results given in figures 2 and 3 compare favourably with the analytical predictions. A noticeable deviation in the time scale (figure 3), which is of no relevance for the sediment composition, seems to be due to an uncertainty in the viscosity data of the liquid.

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